Algebra and Geometry

Classifying Numbers

Type	Definition	Example
Integers	Numbers with no fractional part	$\ldots, -2, -1, 0, 1, 2, \ldots$
Rational Numbers	Fractions of integers: $\frac{m}{n}$, where $n \neq 0$ (includes the set of all integers)	$\frac{4}{3}, 1, -\frac{8}{3}$
Irrational Numbers	Numbers that cannot be expressed as a fraction of integers	$\pi, e, \sin 1, \sqrt{2}, 2+\sqrt{3}$
Real Numbers	The combined set of rational and irrational numbers	All of the above

Factoring Squares and Cubes

$$a^2 - b^2 = (a+b)(a-b)$$

$$a^3 + b^3 = (a+b)\left(a^2 - ab + b^2\right)$$

$$a^3 - b^3 = (a-b)\left(a^2 + ab + b^2\right)$$

Factoring Polynomials

For two numbers p and q with product c and sum b,

$$x^2 + bx + c = (x+p)(x+q)$$

To factor $ax^2 + bx + c$ (where $a \neq 0$), find two numbers p and q whose product is ac and sum is b:

$$ax^2 + bx + c = ax^2 + px + qx + c$$

Then use factoring by grouping by splitting the right side into pairs, each of which shares a common factor.

Operations with Fractions

Assume $b \neq 0$ and $d \neq 0$; for division, assume $c \neq 0$.

$$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd} \qquad \frac{a}{b} \div \frac{c}{d} = \frac{ad}{bc}$$

$$\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd} \qquad \frac{a}{b} - \frac{c}{d} = \frac{ad-bc}{bd}$$

Exponent Laws

If a and b are positive and m and n are real numbers, then

$$a^{m+n} = a^m\, a^n \qquad a^{m-n} = \frac{a^m}{a^n}$$

$$\left(a^m\right)^n = a^{mn} \qquad (ab)^n = a^n\, b^n$$

If n is a positive integer, then

$$\sqrt[n]{a^m} = a^{m/n}$$

Logarithm Laws

Let $b > 0$ and $b \neq 1$. If x and y are positive, then

$$\log_b(xy) = \log_b x + \log_b y \qquad \log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$$

If n is any real number, then

$$\log_b\left(x^n\right) = n \log_b x$$

Also,

$$b^{\log_b x} = x \qquad \log_b\left(b^x\right) = x$$

If $c > 0$ and $c \neq 1$, then

$$\log_b x = \frac{\log_c x}{\log_c b}$$

Absolute Values

$$|a| = \begin{cases} a & a \geqslant 0 \\ -a & a < 0 \end{cases} \qquad \sqrt{a^2} = |a|$$

$$|ab| = |a|\,|b| \qquad \left|\frac{a}{b}\right| = \frac{|a|}{|b|}, \quad b \neq 0$$

If n is a positive integer, then

$$|a^n| = |a|^n$$

Midpoint and Distance Formulas

The **distance** between two points (x_1, y_1) and (x_2, y_2) is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

The **midpoint** is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Functions and the Vertical Line Test

A **function** f maps each element x in the domain D to exactly one element in the range E. A graph represents a function if no vertical line intersects the graph more than once.

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Even and Odd Functions

If a function f has x and $-x$ in its domain, then f is

- **even** if and only if $f(-x) = f(x)$
- **odd** if and only if $f(-x) = -f(x)$

Combinations and Compositions of Functions

$$(f+g)(x) = f(x) + g(x) \qquad (f-g)(x) = f(x) - g(x)$$

$$(fg)(x) = f(x) \cdot g(x) \qquad \left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, \quad g(x) \neq 0$$

If x is in the domain of g, and $g(x)$ is in the domain of f, then

$$(f \circ g)(x) = f\left(g(x)\right)$$

Linear Functions

If a line has **slope** m (rise/run) and y-intercept $(0, b)$, then

$$y = mx + b$$

Alternatively, if (x_0, y_0) is a point on the line, then

$$y - y_0 = m\left(x - x_0\right)$$

Two lines are **parallel** if they share the same slope; two lines are **perpendicular** if their slopes are negative reciprocals of each other.

Solving Systems of Linear Equations

For a system of two independent linear equations, use either method:

- **Substitution.** In one equation, express one variable in terms of the other; substitute this expression into the second equation.
- **Elimination.** Rewrite each equation in the form $ax + by = c$ and align both equations vertically. Multiply one equation by a factor so that both equations have coefficients of x or y that are equal in magnitude. Then add or subtract the equations as appropriate to eliminate one variable.

Quadratics

Assume $a \neq 0$.

Quadratic Form	Equation and Description
General Form	$y = ax^2 + bx + c$, where $(0, c)$ is the y-intercept
Vertex Form	$y = a(x-h)^2 + k$, where $h = -\frac{b}{2a}$ (may be obtained by completing the square)
Factored Form	$y = a(x-p)(x-q)$, where p and q are zeros

If $a > 0$, then the parabola opens upward and the vertex is the lowest point; if $a < 0$, then the parabola opens downward and the vertex is the highest point.

Quadratic Formula

The solutions to $ax^2 + bx + c = 0$, where $a \neq 0$, are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Inverse Functions

A function f is **one-to-one** if and only if

$$f\left(x_1\right) \neq f\left(x_2\right) \qquad \text{for } x_1 \neq x_2$$

The graphs of $y = f(x)$ and $y = f^{-1}(x)$ are reflections across the line $y = x$, and

$$f\left(f^{-1}(x)\right) = x \qquad \text{and} \qquad f^{-1}\left(f(x)\right) = x$$

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Circles and Circular Sectors

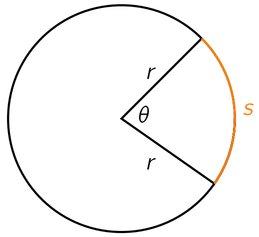
For a whole circle of radius r, area A, and circumference C,

$$A = \pi r^2 \qquad C = 2\pi r$$

For a circular sector of area A and arc length s,

$$A = \tfrac{1}{2}\theta r^2 \qquad s = r\theta$$

where θ is expressed in radians (π rad = 180°).



Conic Sections

Graph Description	Equation
Circle with center (h, k) and radius r	$(x-h)^2 + (y-k)^2 = r^2$
Ellipse with center (h, k), width $2a$, and height $2b$	$\dfrac{(x-h)^2}{a^2} + \dfrac{(y-k)^2}{b^2} = 1$
Hyperbola that opens horizontally with x-intercepts $(-a, 0)$ and $(a, 0)$	$\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$
Hyperbola that opens vertically with y-intercepts $(0, -a)$ and $(0, a)$	$\dfrac{y^2}{a^2} - \dfrac{x^2}{b^2} = 1$

Translating Graphs of Functions

Let $c > 0$.

New Function	Transformation
$y = f(x) + c$	$y = f(x)$ shifted up c units
$y = f(x) - c$	$y = f(x)$ shifted down c units
$y = f(x + c)$	$y = f(x)$ shifted left c units
$y = f(x - c)$	$y = f(x)$ shifted right c units

Dilating Graphs of Functions

Let $a > 1$.

New Function	Transformation
$y = af(x)$	Stretch $y = f(x)$ vertically by a factor of a
$y = \dfrac{1}{a}f(x)$	Compress $y = f(x)$ vertically by a factor of a
$y = f(ax)$	Compress $y = f(x)$ horizontally by a factor of a
$y = f\left(\dfrac{x}{a}\right)$	Stretch $y = f(x)$ horizontally by a factor of a

Sigma Notation

Let a_i and b_i be real numbers for integers $m \leqslant i \leqslant n$, and let c be a constant.

$$\sum_{i=m}^{n} a_i = a_m + a_{m+1} + a_{m+2} + \cdots + a_{n-1} + a_n$$

$$\sum_{i=m}^{n} (a_i + b_i) = \sum_{i=m}^{n} a_i + \sum_{i=m}^{n} b_i$$

$$\sum_{i=m}^{n} (a_i - b_i) = \sum_{i=m}^{n} a_i - \sum_{i=m}^{n} b_i$$

$$\sum_{i=m}^{n} c\, a_i = c \sum_{i=m}^{n} a_i$$

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Summation Formulas

Let c be a constant and n be a positive integer.

$$\sum_{i=1}^{n} c = c\,n \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$$

Properties of Functions

Function	**Domain**	**Range**
$y = c$ for any real c	$(-\infty, \infty)$	$\{c\}$
$y = mx + b$ for $m \neq 0$	$(-\infty, \infty)$	$(-\infty, \infty)$
$y = x^2$	$(-\infty, \infty)$	$[0, \infty)$
$y = \frac{1}{x}$	$(-\infty, 0) \cup (0, \infty)$	$(-\infty, 0) \cup (0, \infty)$
$y = b^x$, $b > 0$ and $b \neq 1$	$(-\infty, \infty)$	$(0, \infty)$
$y = \log_b x$, $b > 0$ and $b \neq 1$	$(0, \infty)$	$(-\infty, \infty)$
$y = \lvert x \rvert$	$(-\infty, \infty)$	$[0, \infty)$
$y = \sqrt[n]{x}$ for positive even n	$[0, \infty)$	$[0, \infty)$
$y = \sqrt[n]{x}$ for positive odd n	$(-\infty, \infty)$	$(-\infty, \infty)$